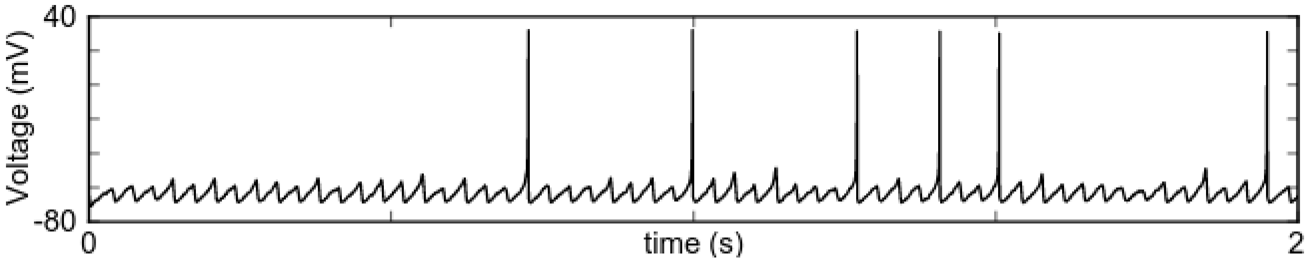
E/I balance – oscillatory variable population activity

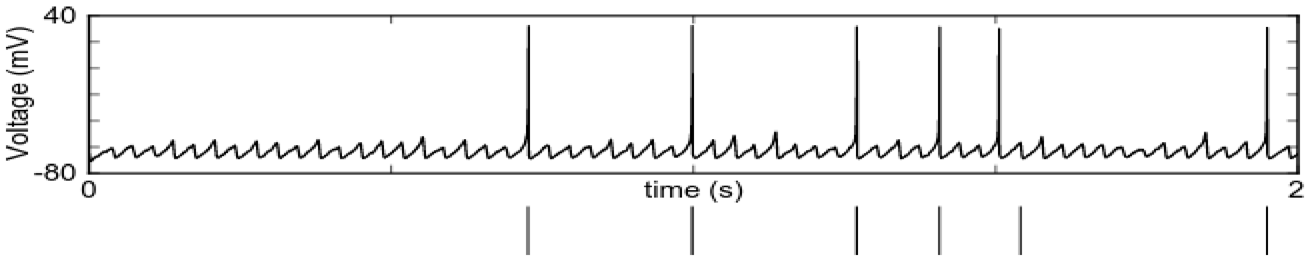
- From individual neurons to a population
- From individual spikes to a population spike rate
- Step 1: self-excitatory/-inhibitory population. Bistablity.
- Step 2: self-excitatory population with linear adaptation.
- Step 3: self-excitatory population with non-linear adaptation.
- Step 4: Wilson-Cowan model.

From individual neurons to a population



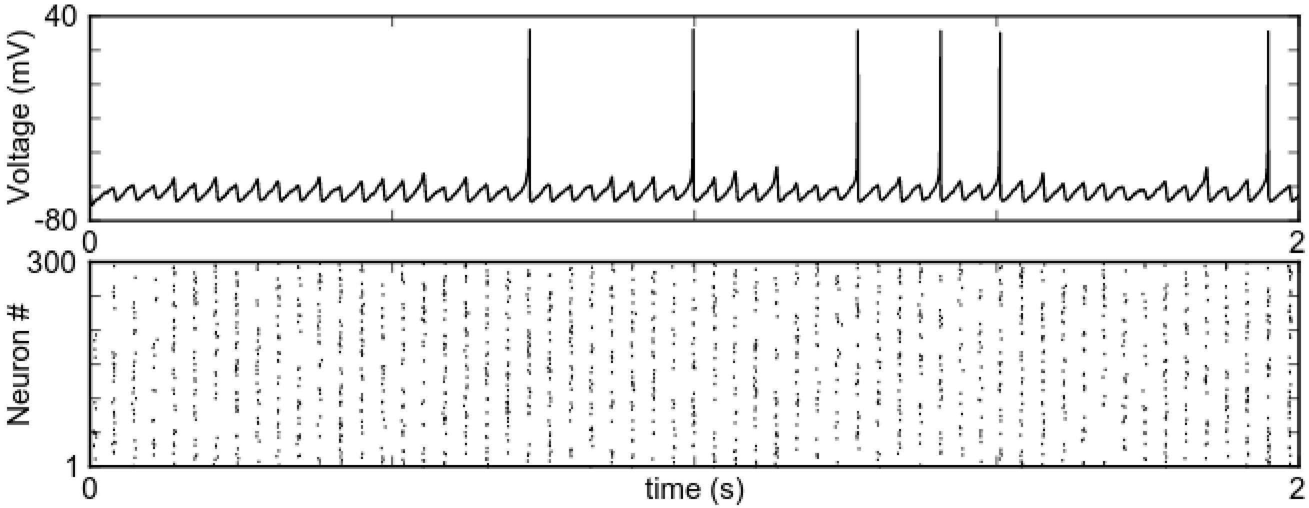
We record an individual neuron...
BUT
it is embedded into network.

From individual neurons to a population



Let mark every spike by just vertical bar.

From individual neurons to a population



Raster plot indicates
an activity of the population.

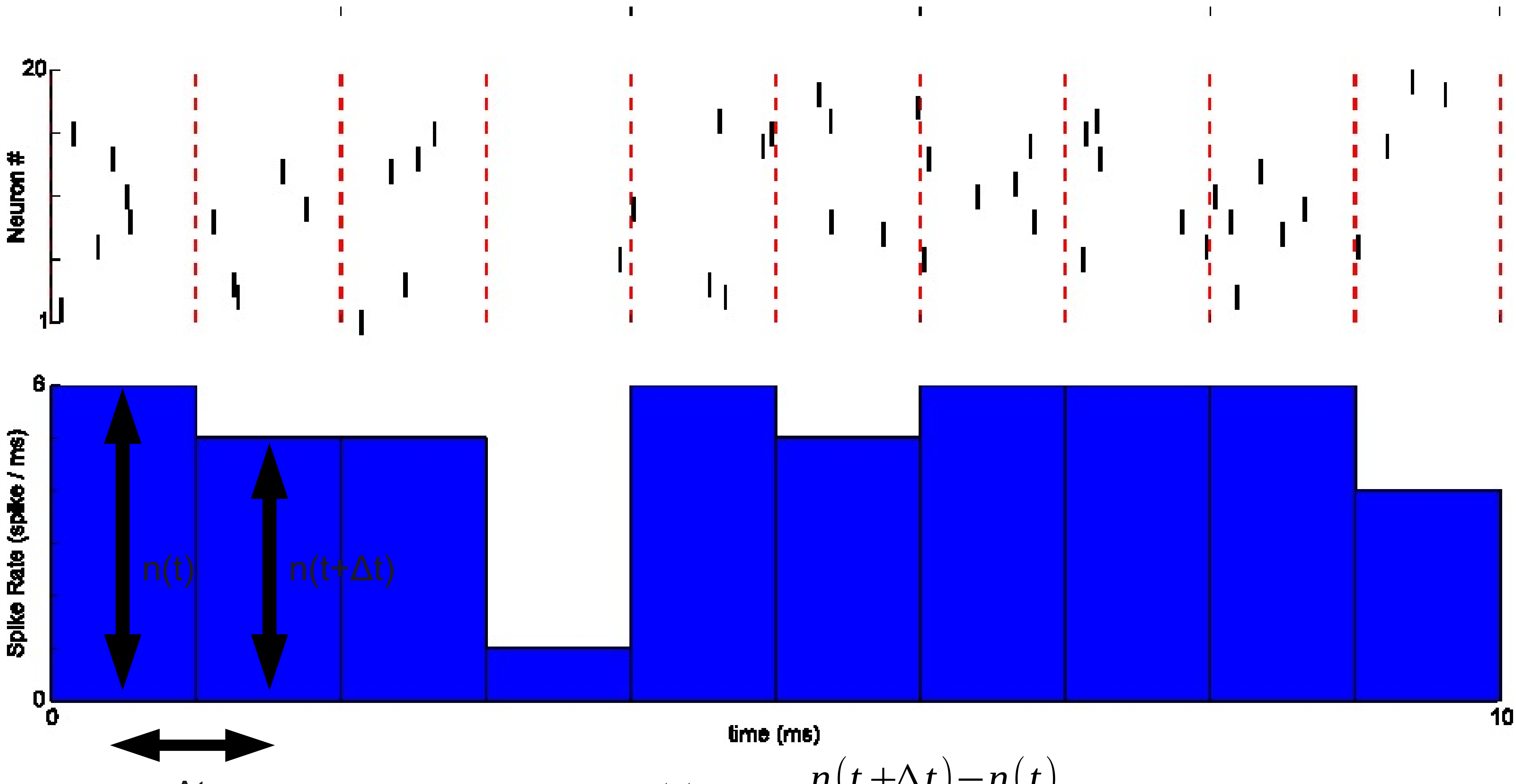
Assumptions:

We assume that all neurons are similar. So we don't care about individual neurons anymore.

We assume that connection into the population are similar. So we don't care about any possible subpopulations of neurons inside the population.

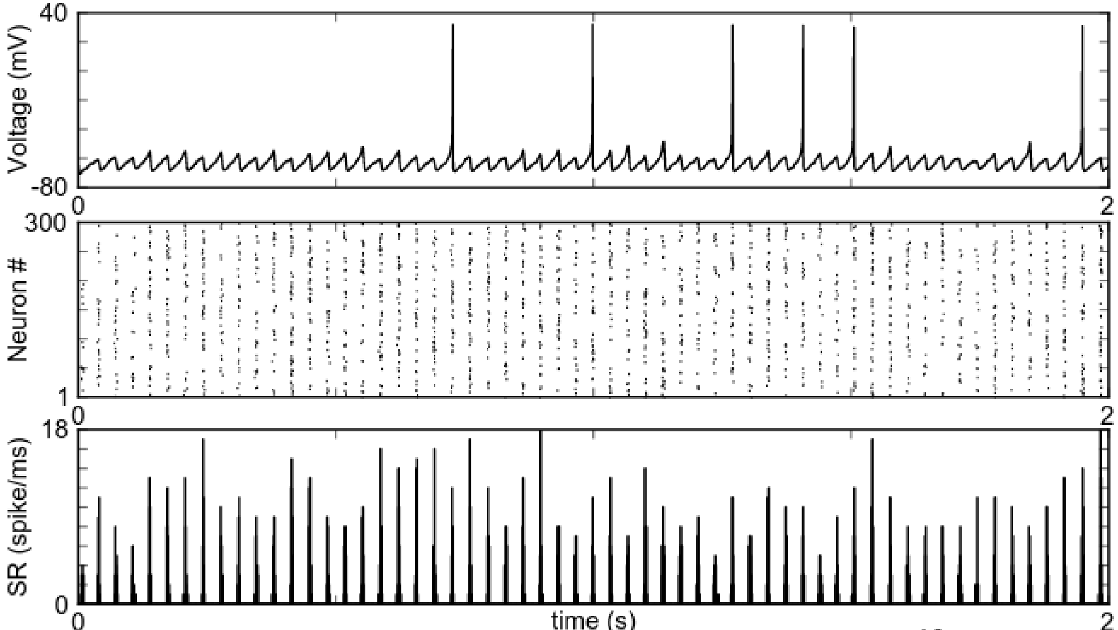
We assume that all neurons within the population have around same both excitatory and inhibitory inputs.

From individual spikes to a population spike rate

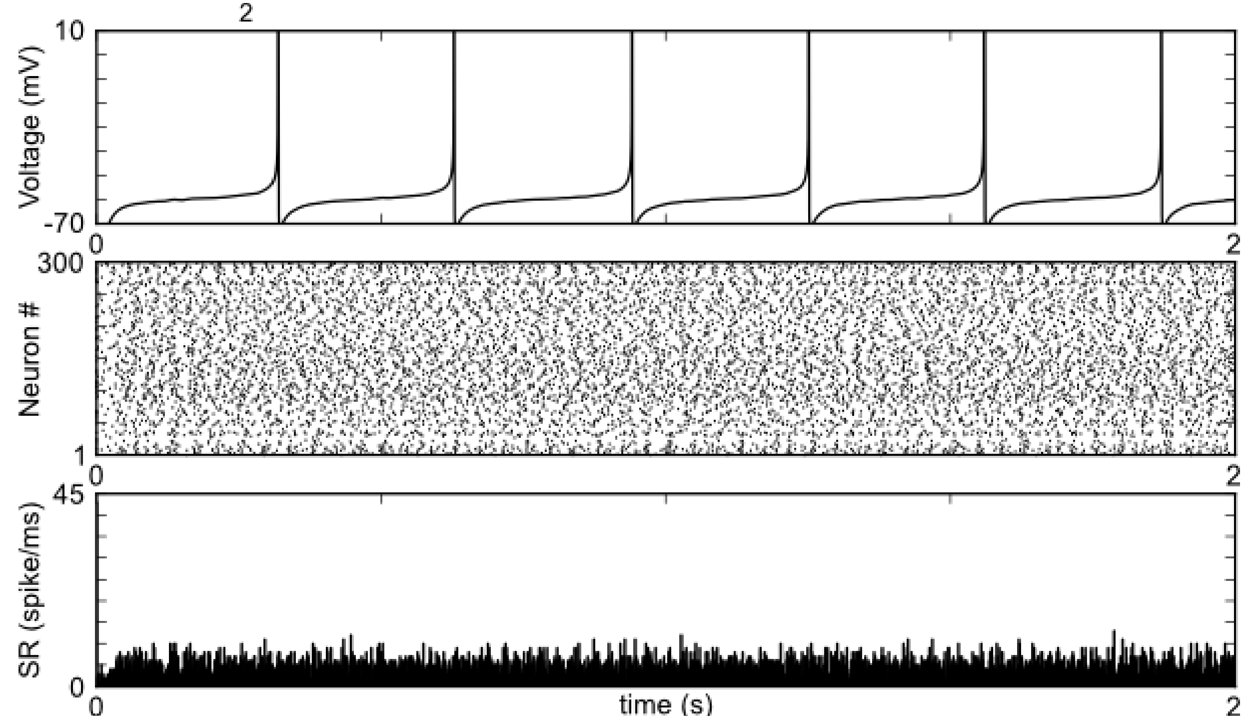


Spike rate = $x(t) = \lim_{\Delta t \to 0} \frac{n(t+\Delta t) - n(t)}{\Delta t}$

What can we see in Population Spike Rate?

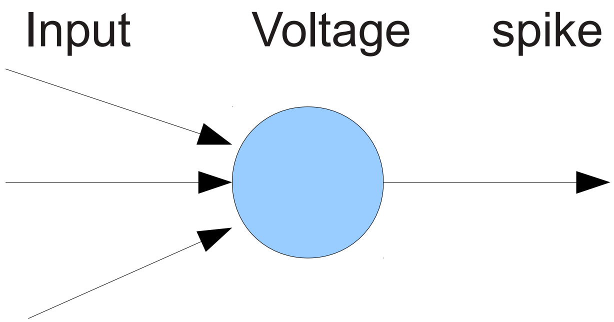


Population SR has periodical solution,
when population fires in synchrony.

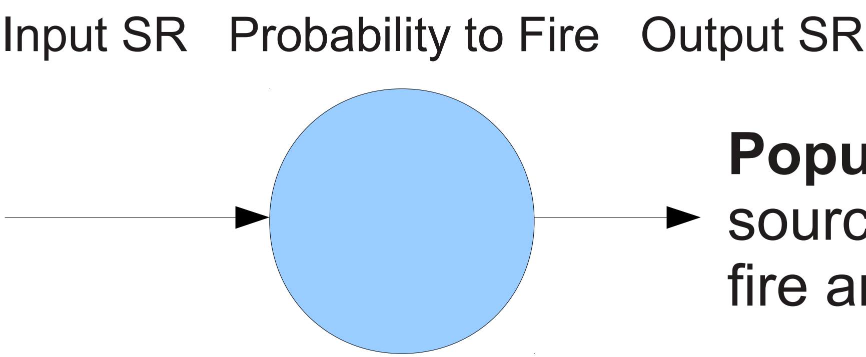


Population SR is constant,
when population is in asynchronous
regime.

From individual neurons to a population



Neuron receives multiple inputs, processes them by changing membrane potential (i.e. voltage) and releases a spikes.

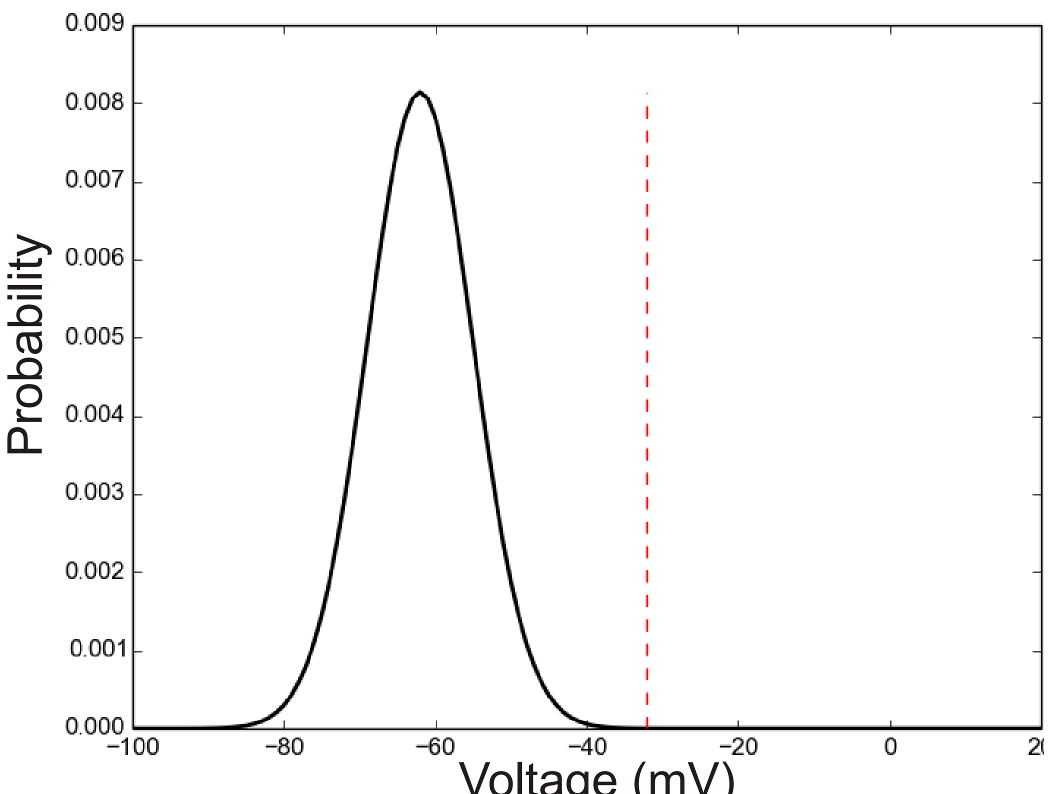


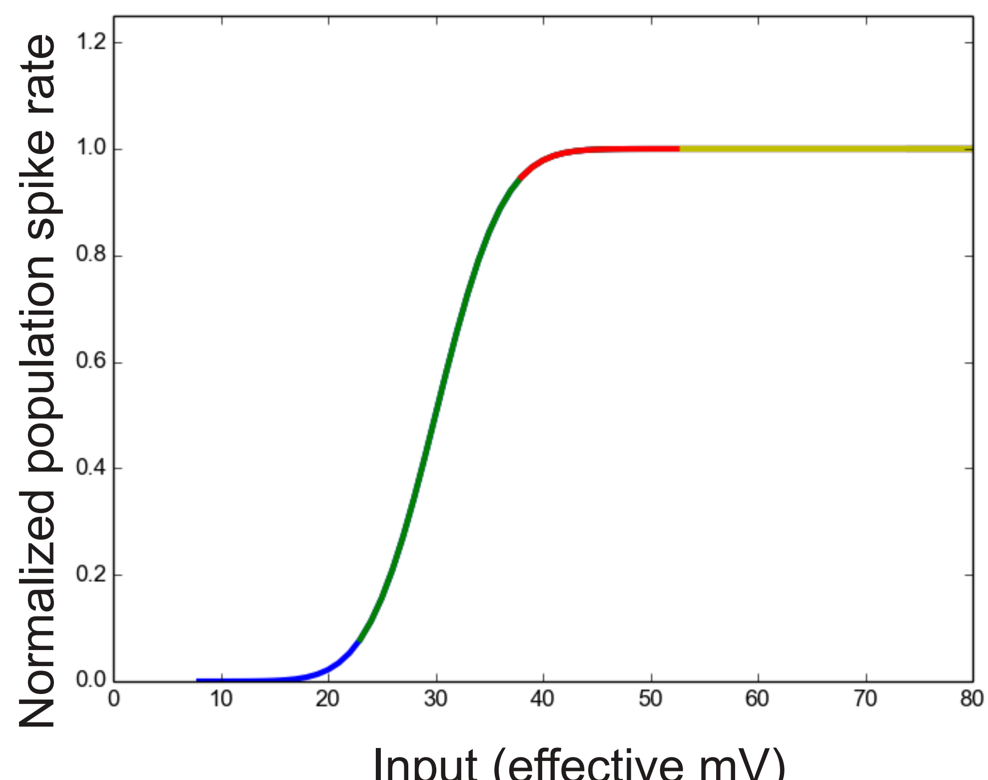
Population receives spike rate inputs from different sources, processes them by changing probability to fire and forms output spike rate.

From individual neurons to a population

Let every neuron within population has subthreshold symmetrical noise (i.e. neuron doesn't fire spontaneously). This noise forms normal distribution of voltages within the population (black). Increase of input excitatory firing rate shift this curve toward the threshold (color curves).

This forms an output population firing rate as a commutative error function (erf).
We usually simplify it to sigmoid function: $$s(x)=\frac{1}{1+\exp\left(-\frac{x-\theta}{k}\right)}$$





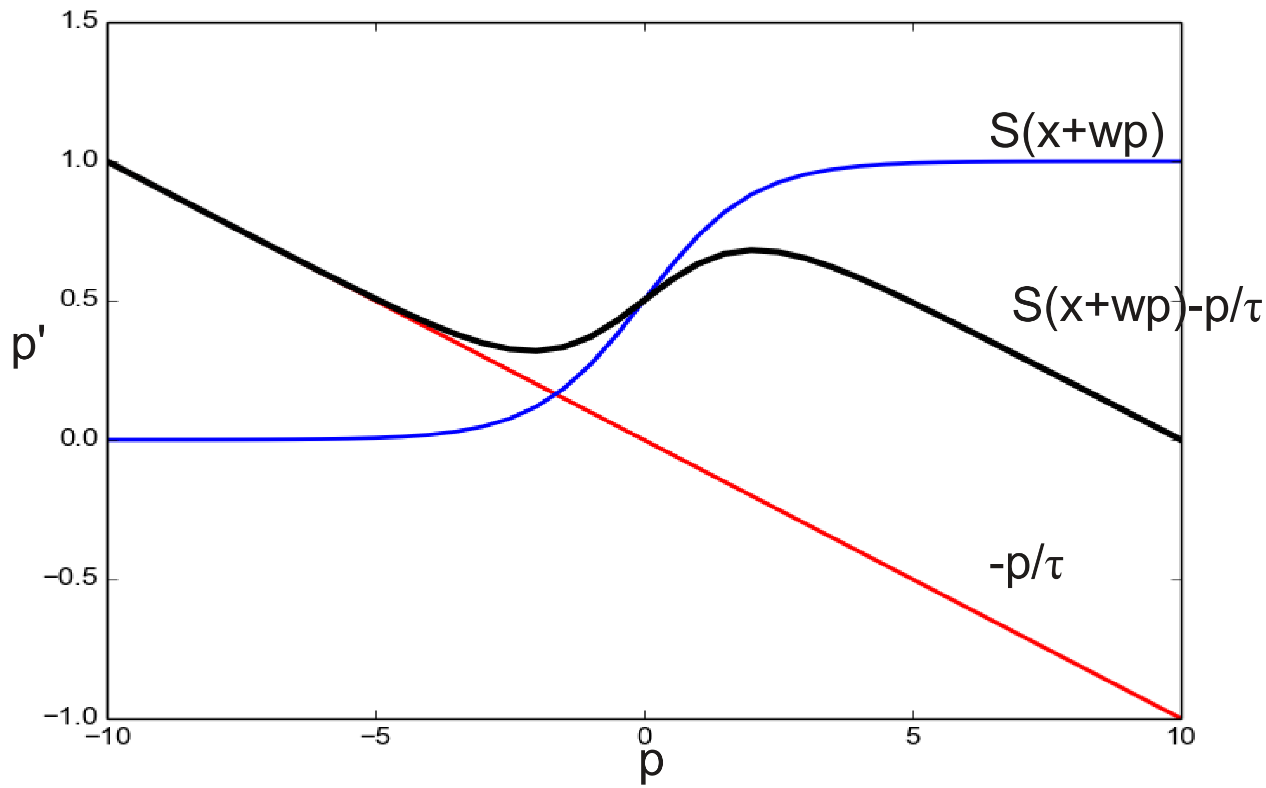
Step 1: self-excitatory or self-inhibitory population. Bistablity.

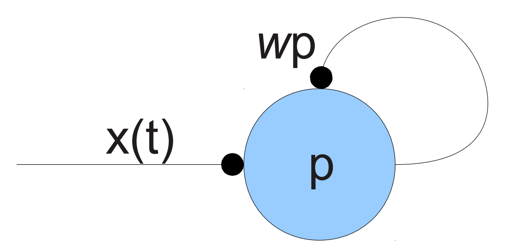
Because without external input population decay to resting potential with time constant τ, an output firing rate can be defined as ordinary differential equation.
Consider self-excitatory population.

$$\frac{dp}{dt} = S(x(t) + wp - \theta) - \frac{p}{\tau}$$

where, S(x(t)) is the sigmoid function from input spike rate x(t), *w* is feedback synaptic weight, *Θ* is threshold and *p* is output SR.

Let consider phase plane *p* vs *p'*





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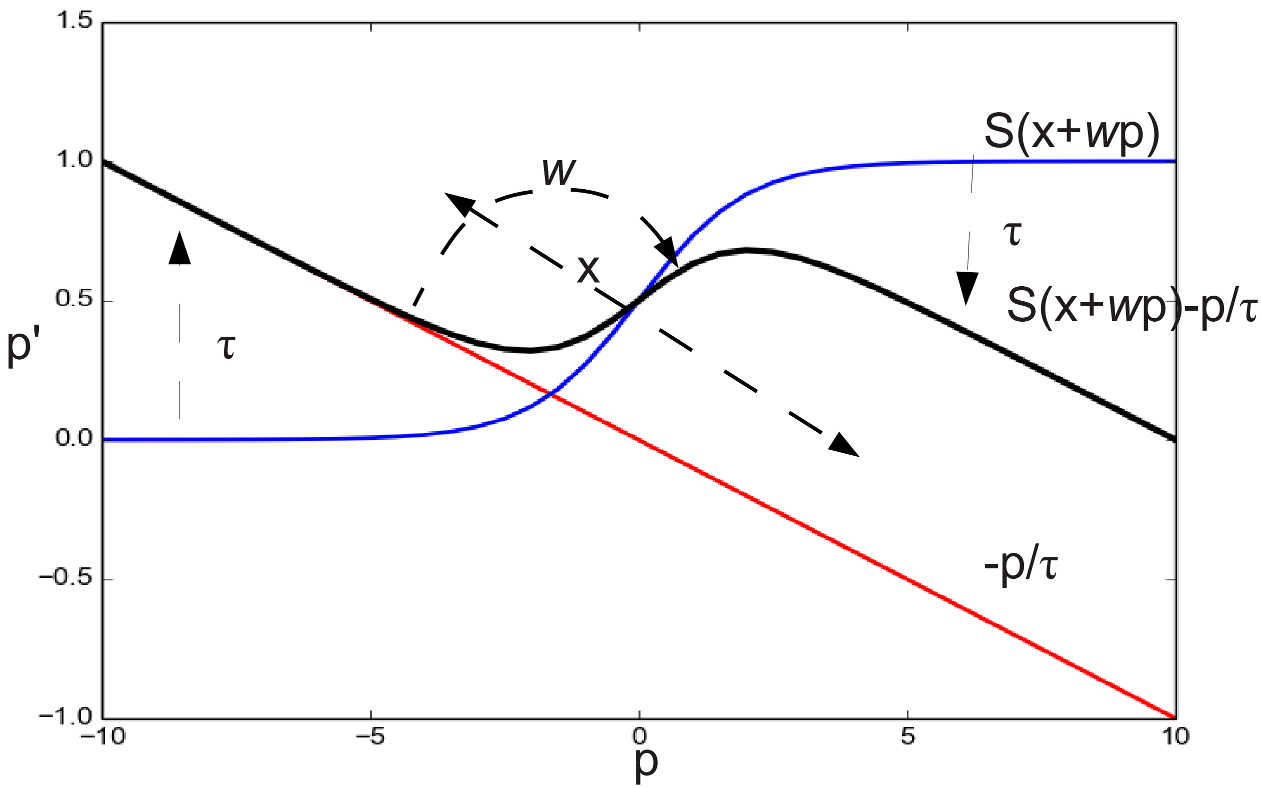

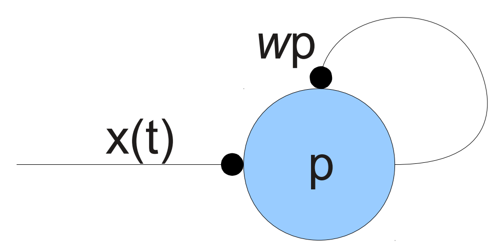

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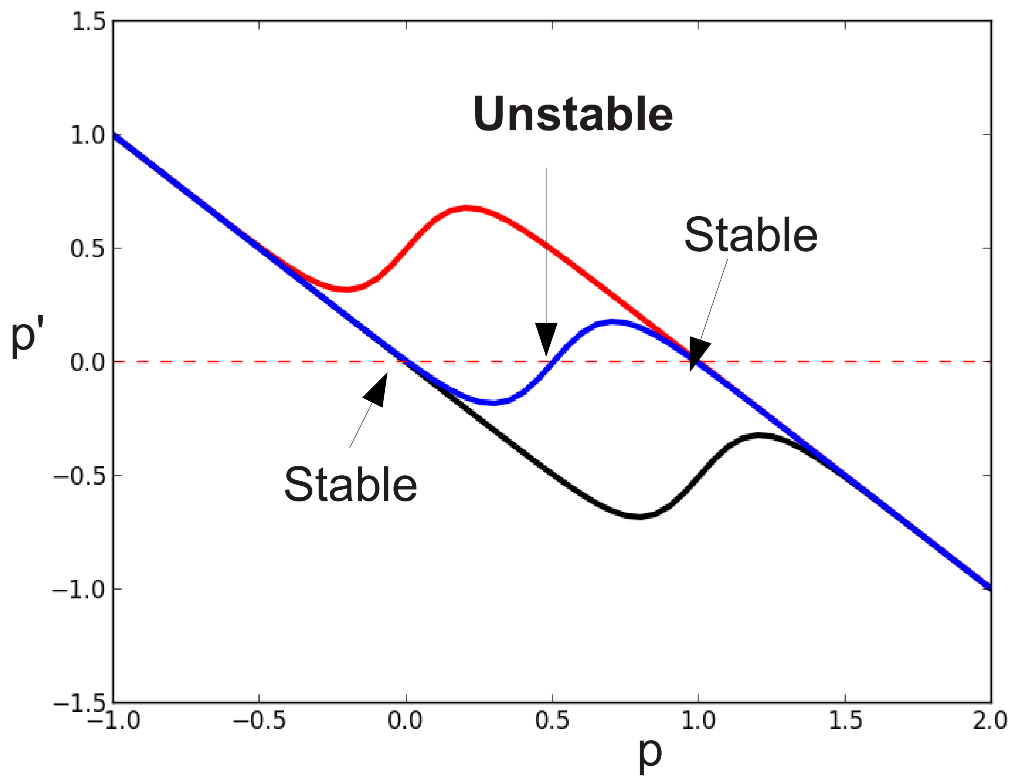

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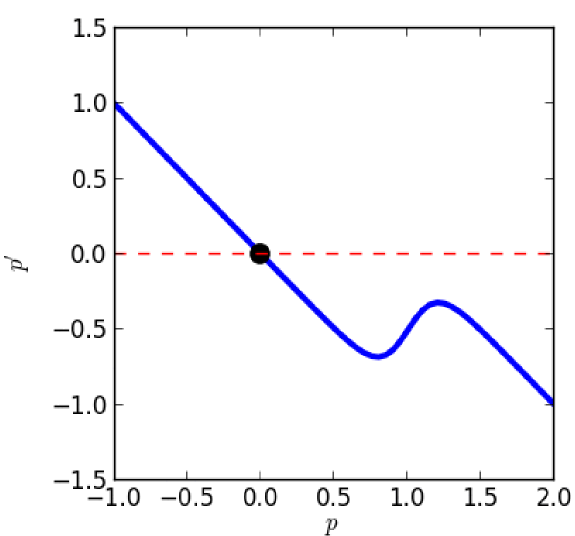
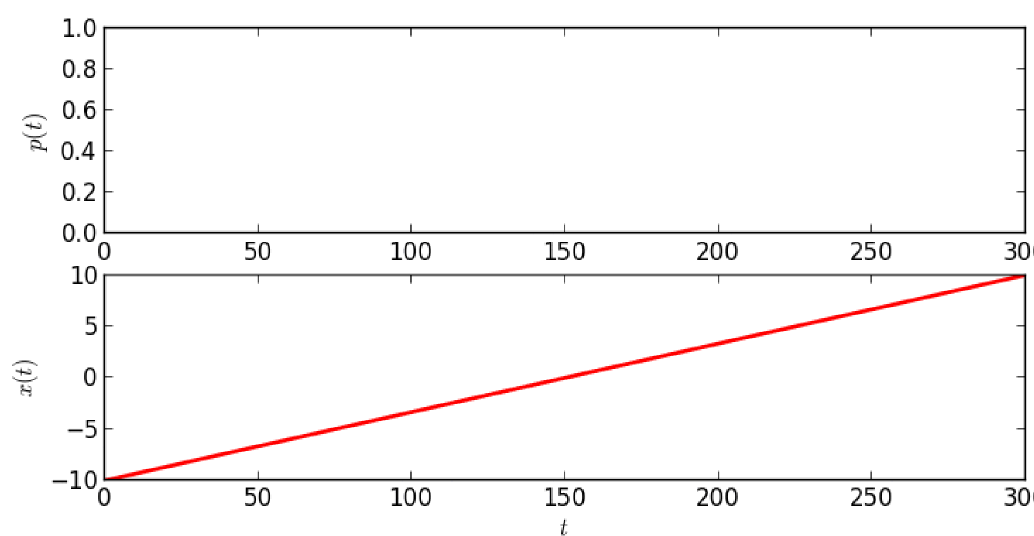
Way UP





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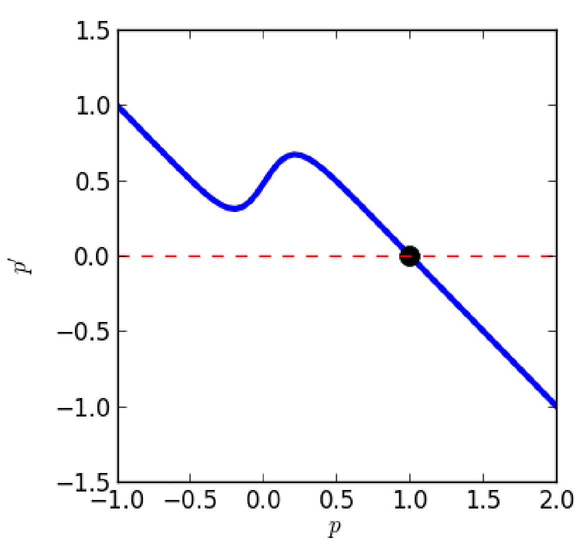
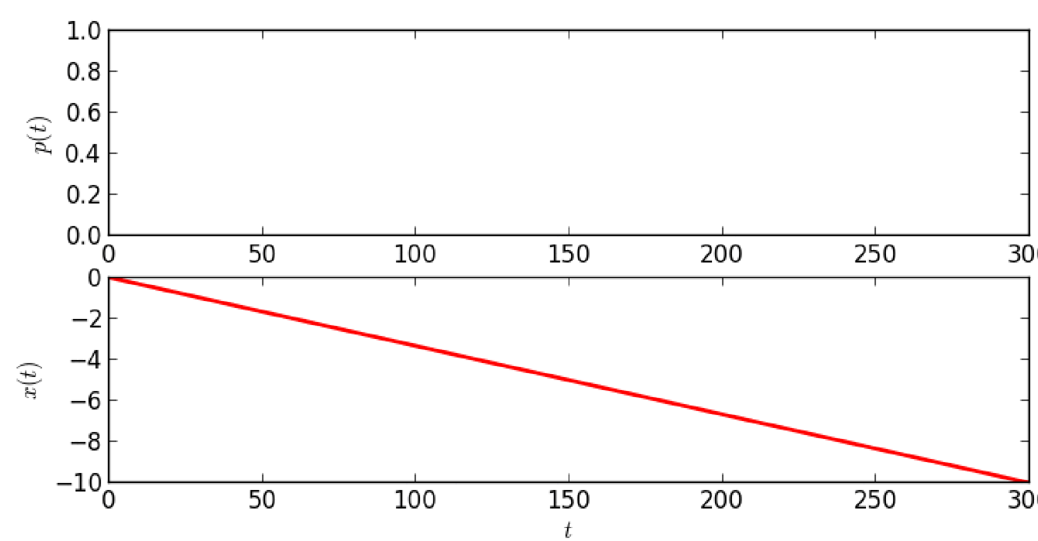
Way Down





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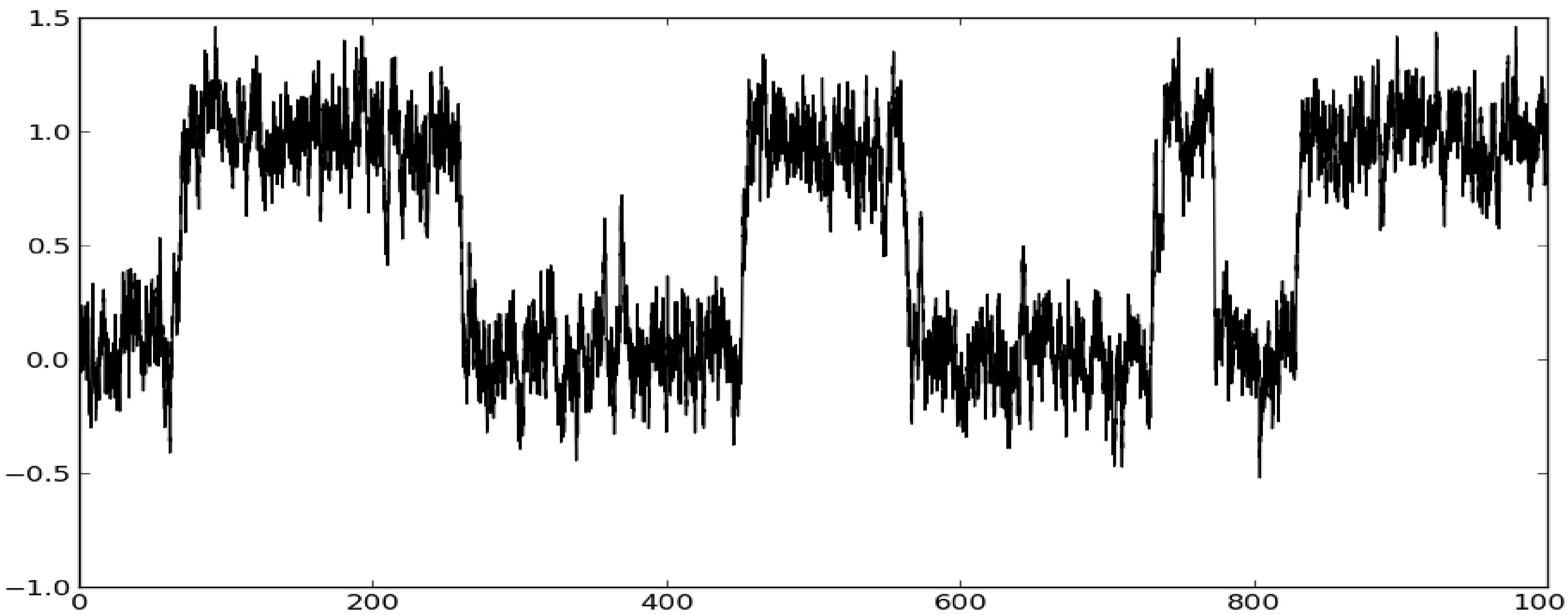
Noise in bistable regime



Step 2: self-excitatory population with linear adaptation

$$\frac{dp}{dt} = S(x(t) + wp) - a - \frac{p}{\tau}$$

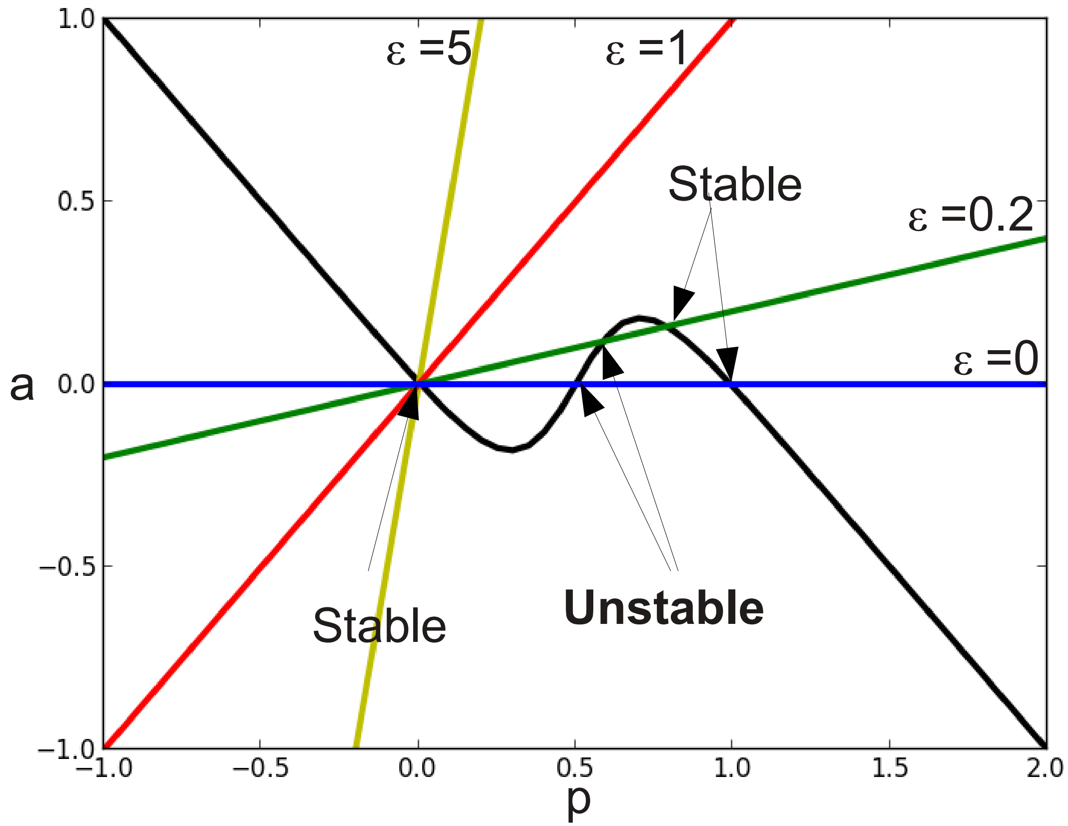
$$\frac{da}{dt} = \frac{\varepsilon p - a}{\tau_a}$$

where, *a* is adaptation variable, ε is adaptation gain and τ_a is adaptation time constant. Usually $\tau_a >> \tau$.

If ε is equal to the zero, we will have the same one equation as we had before, because a(t)=const=0

So ε actually rotates the ox-ordinate.

We have to consider p a phase plane.



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$$\frac{dp}{dt}=S(x(t)+wp)-a-\frac{p}{\tau}$$

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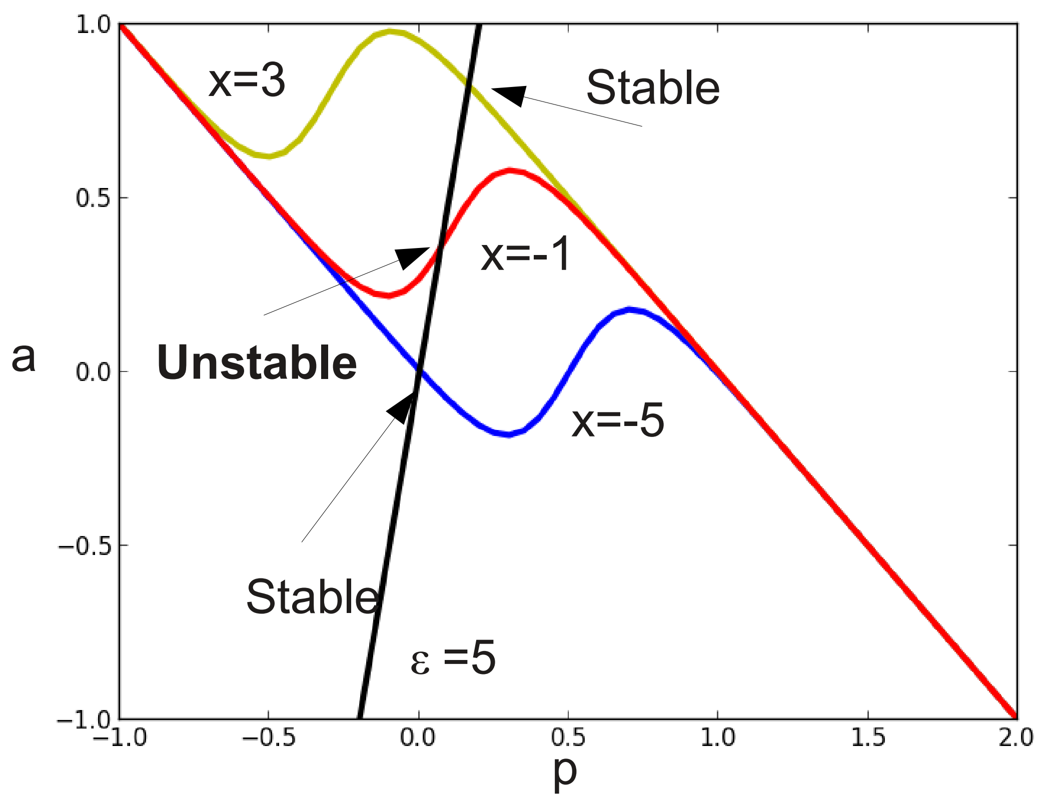

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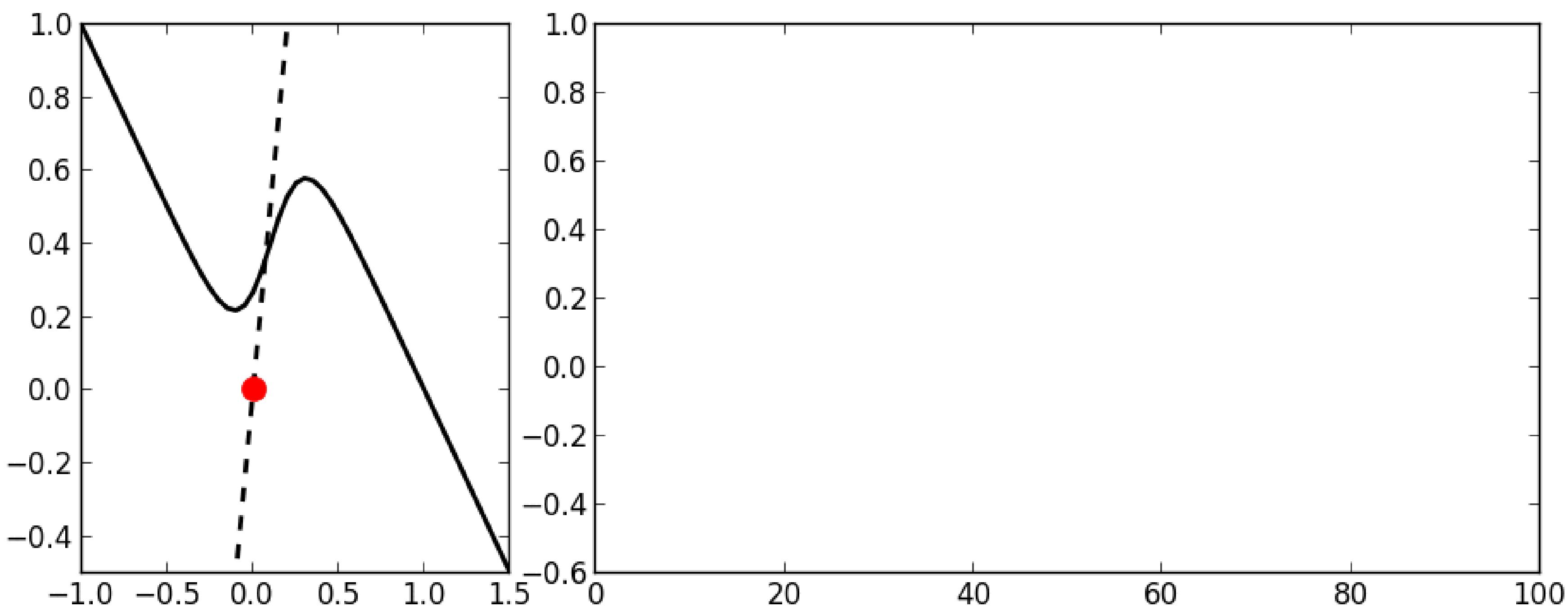
Slow adaptation τ_a =100 $\tau=1$.



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$$\frac{dp}{dt} = S(x(t) + wp) - a - \frac{p}{\tau}$$

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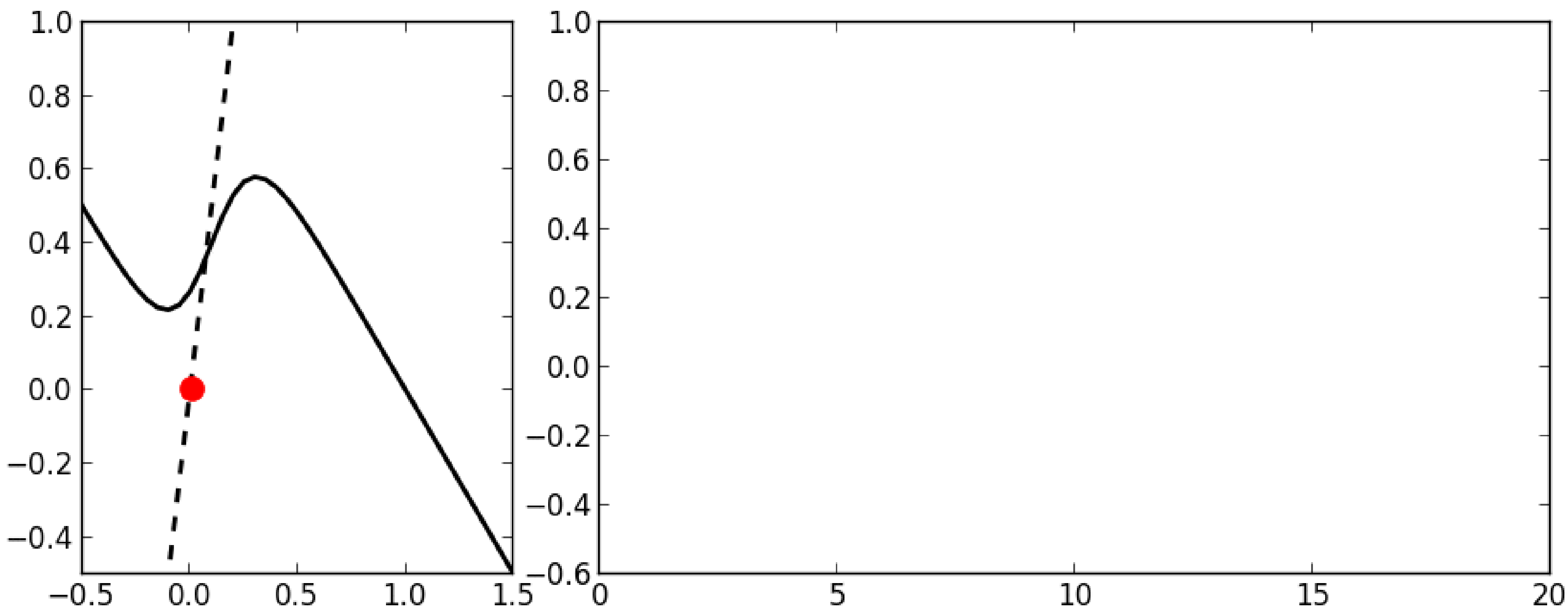
Fast adaptation τ_a =10 $\tau=1$



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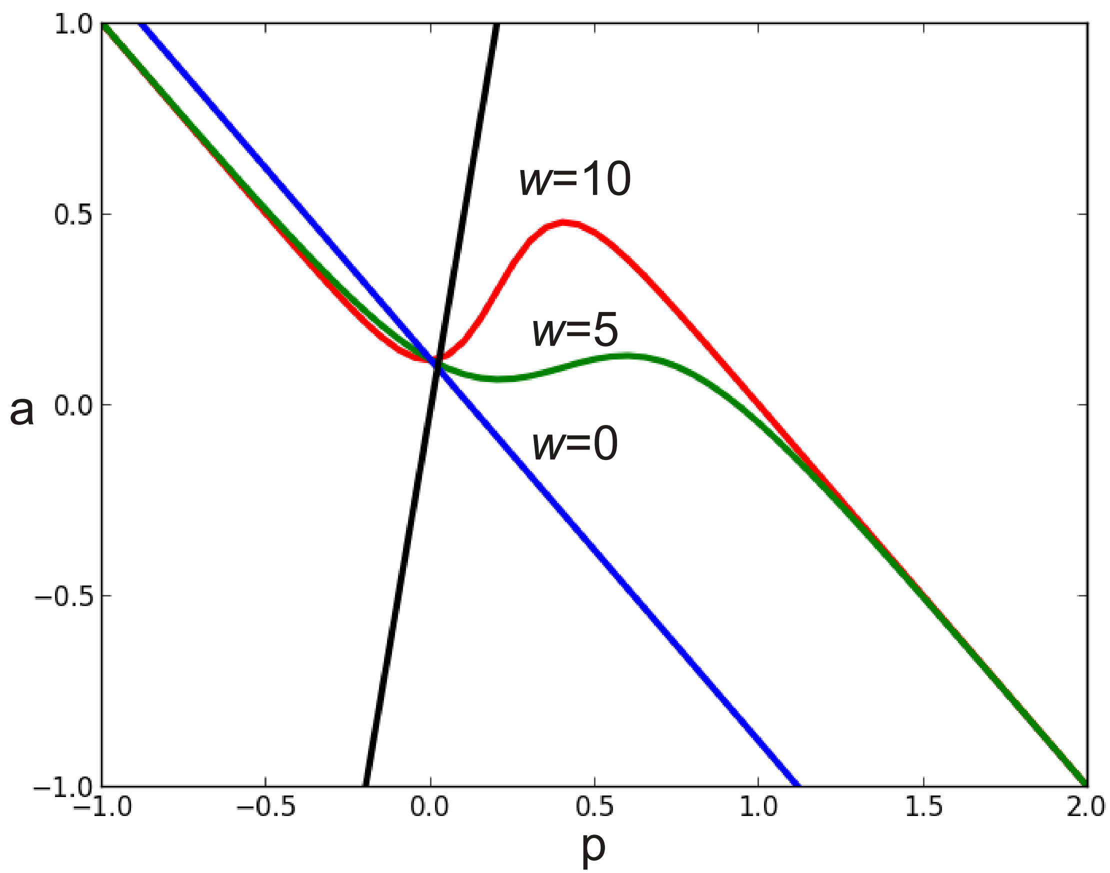
$$\frac{dp}{dt} = S(x(t) + wp) - a - \frac{p}{\tau}$$

$$\frac{da}{dt} = \frac{\varepsilon p - a}{\tau_a}$$

When do these oscillations appear? When excitation and ~~inhibition~~ adaptation are in perfect balances



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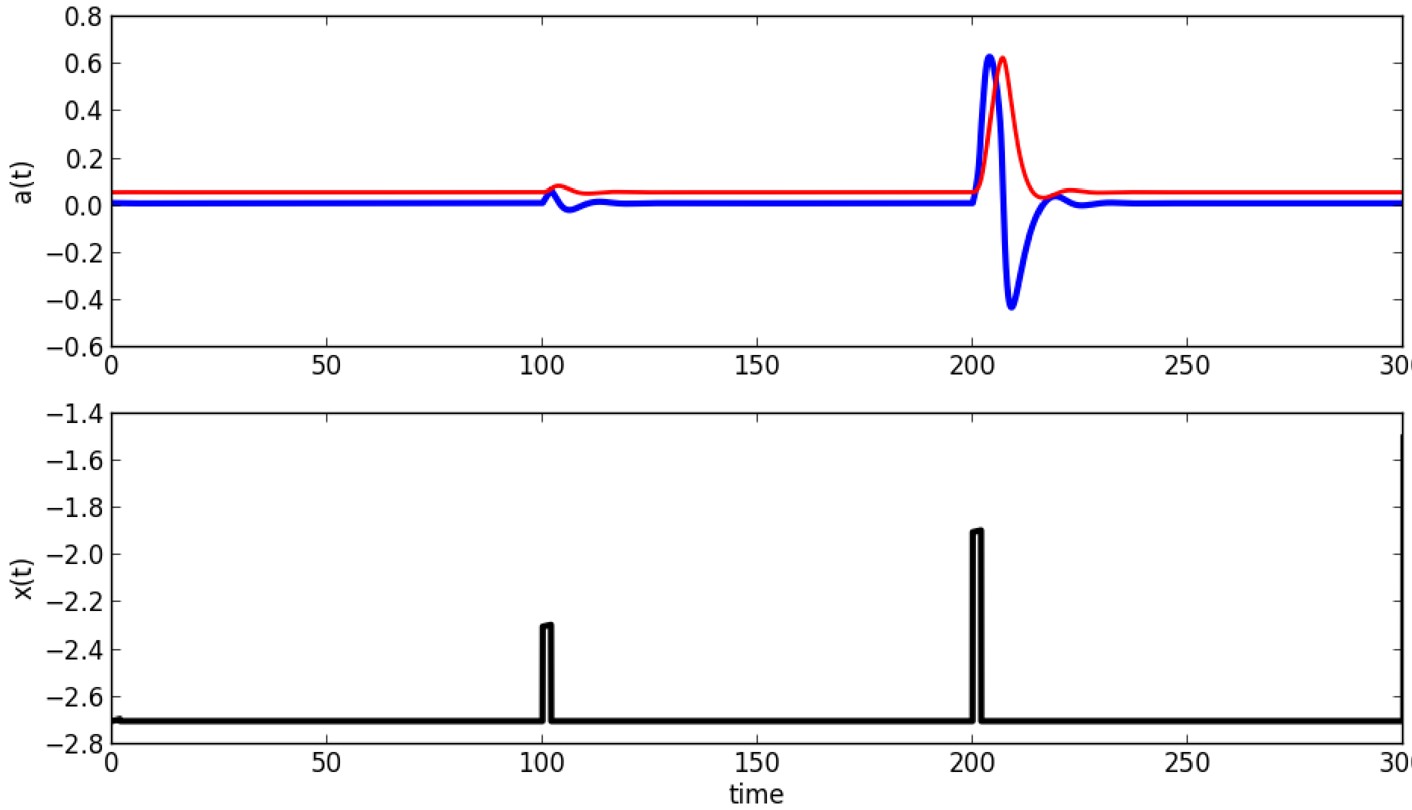
Subthreshold oscillations



Step 3: self-excitatory population with non-linear adaptation. Case II

$$\frac{dp}{dt} = S(x(t) + wp) - a - \frac{p}{\tau}$$

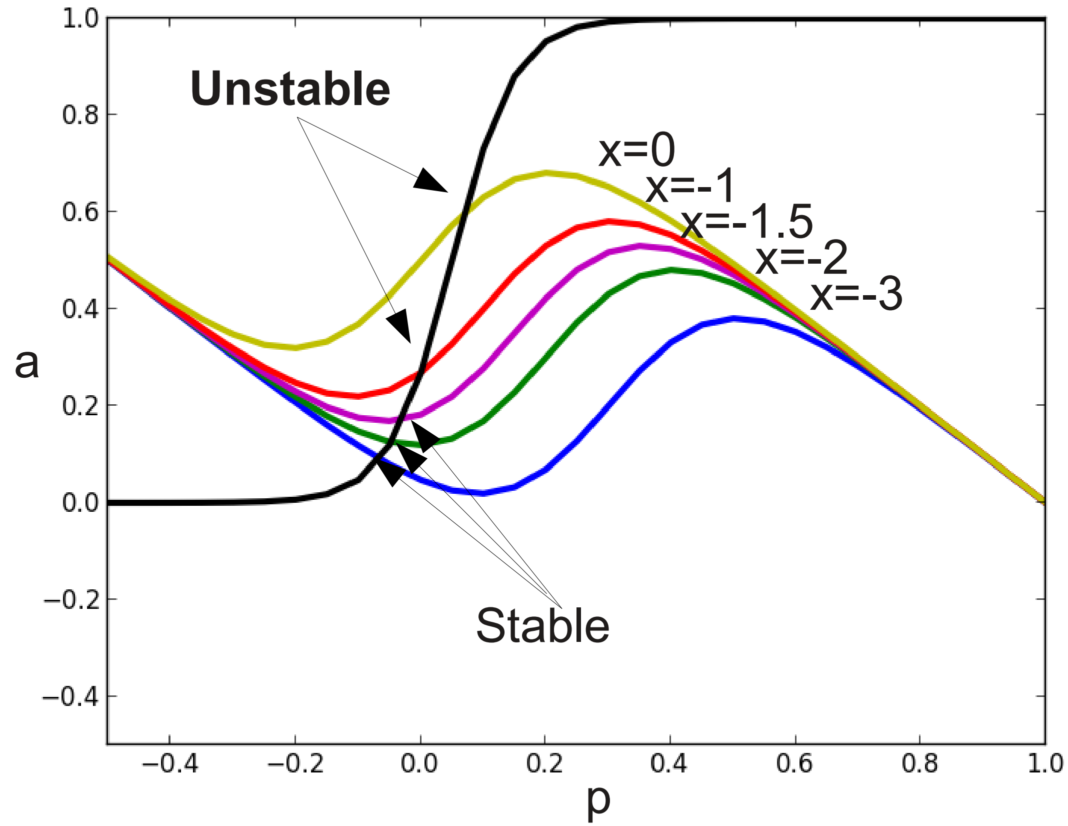
$$\frac{da}{dt} = \frac{S(\varepsilon p - \xi) - a}{\tau_a}$$

What if adaptation non-linear, say same sigmoid function?

Case II

If adaptation is steep enough and doesn't shifted so far.....

ε=20,ξ=1



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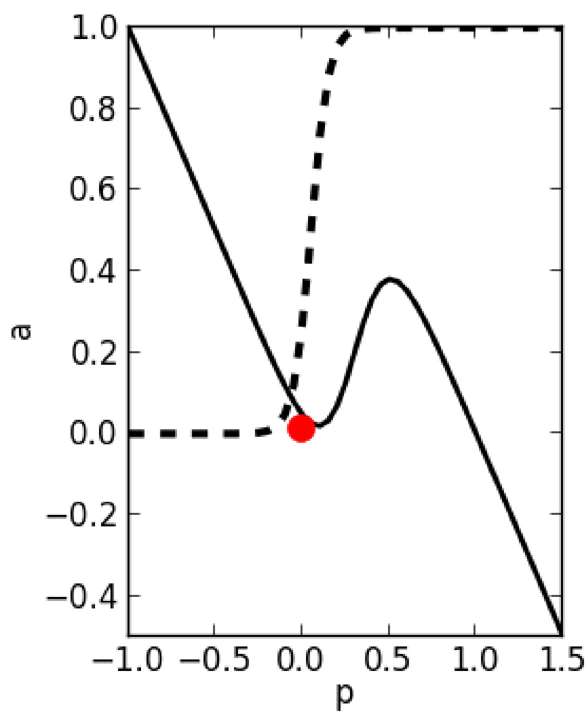

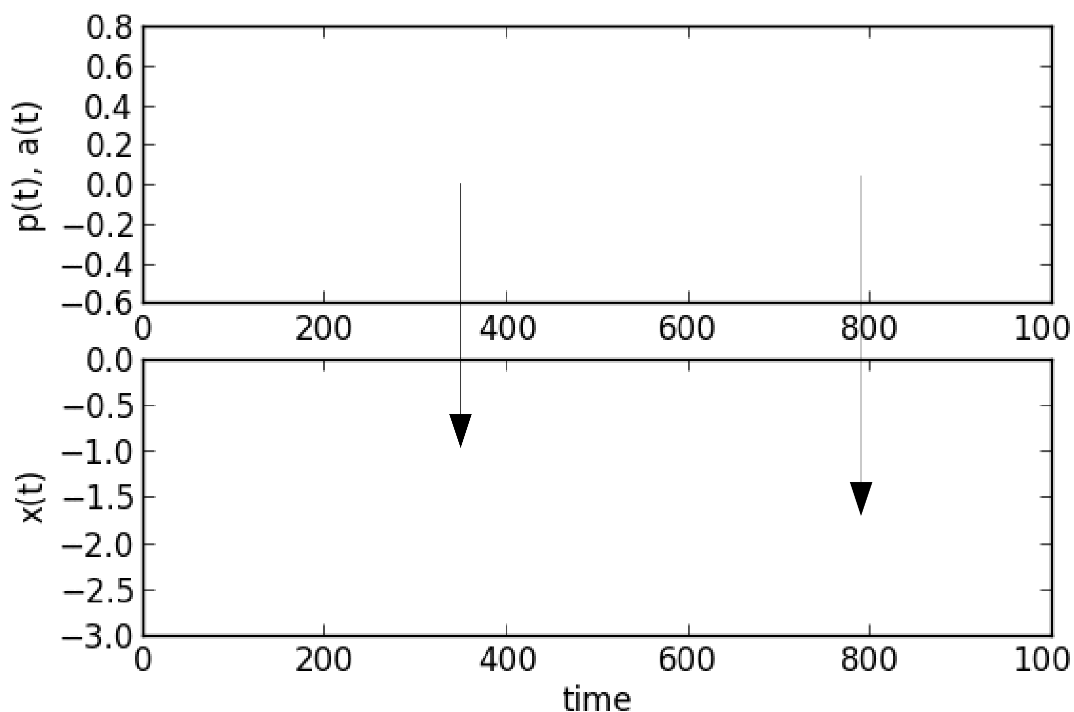

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Case I

If adaptation is very steep and shifted far away.....

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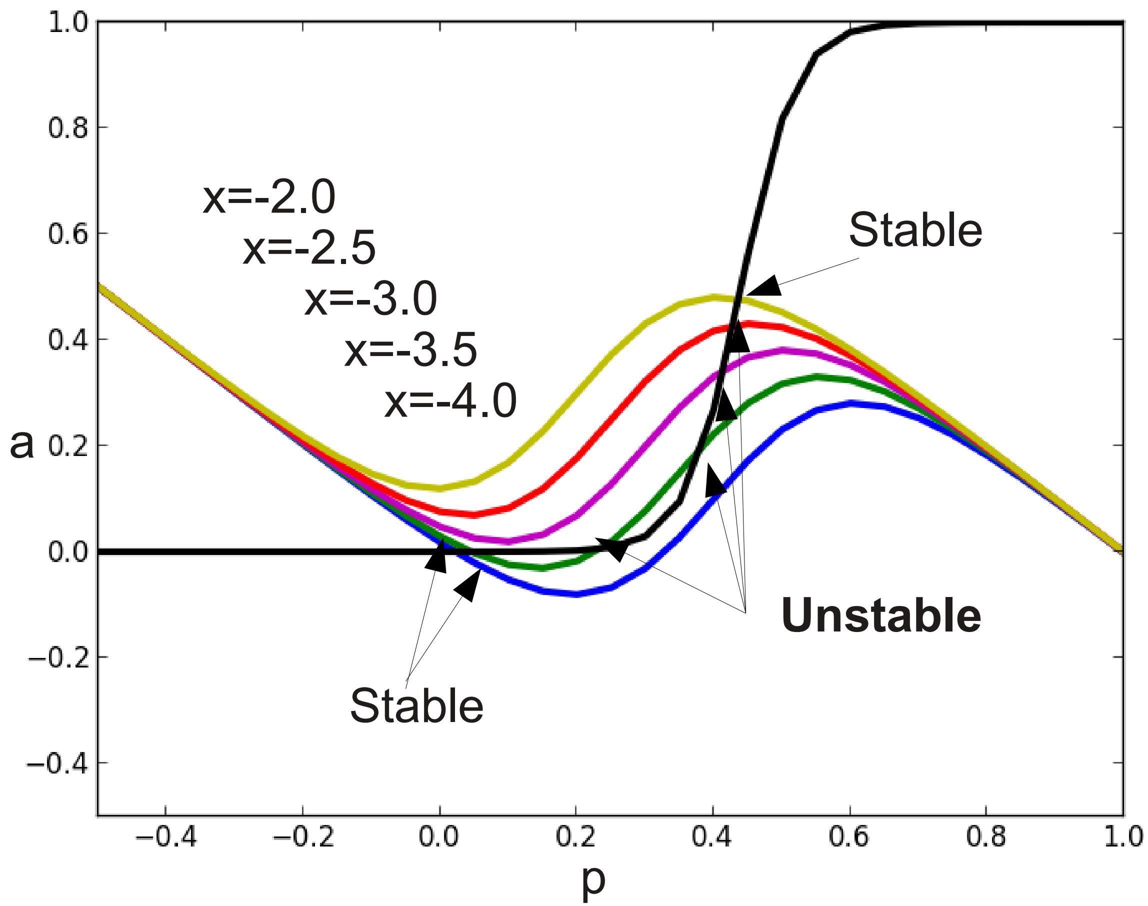

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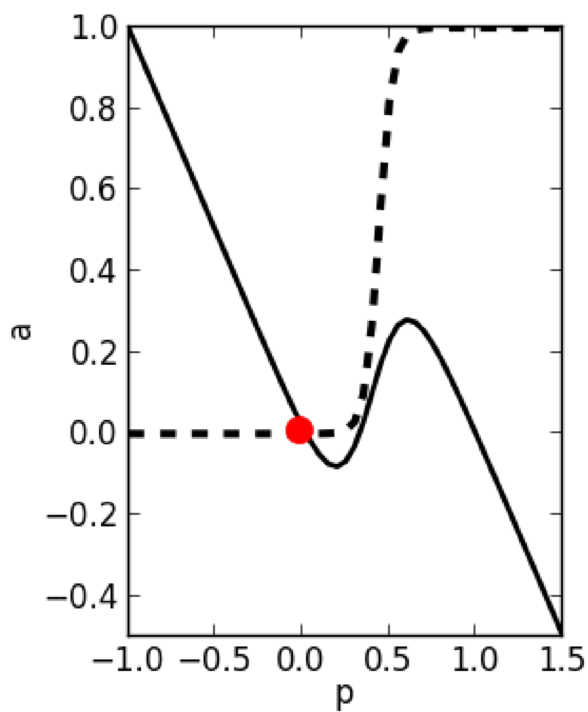

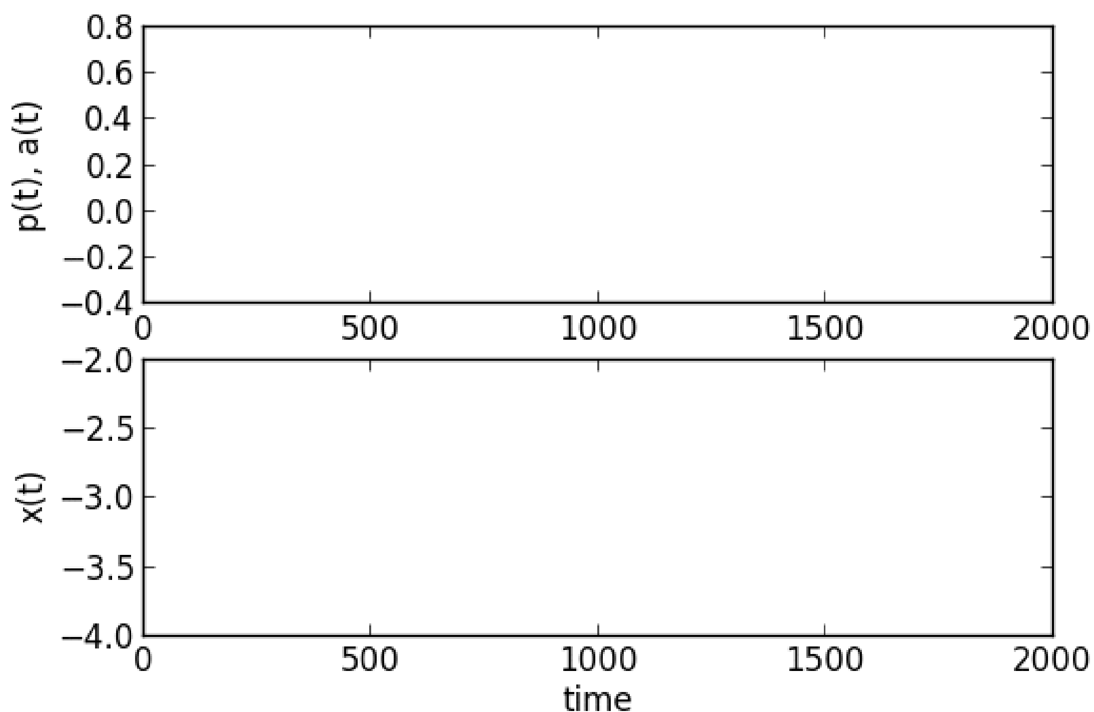

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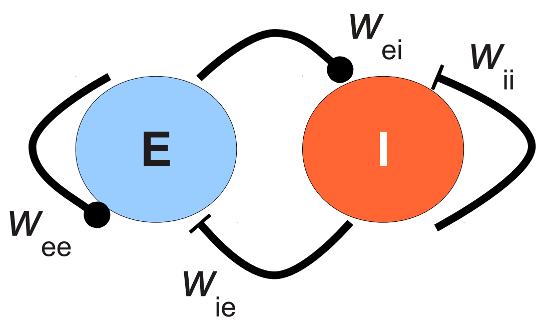


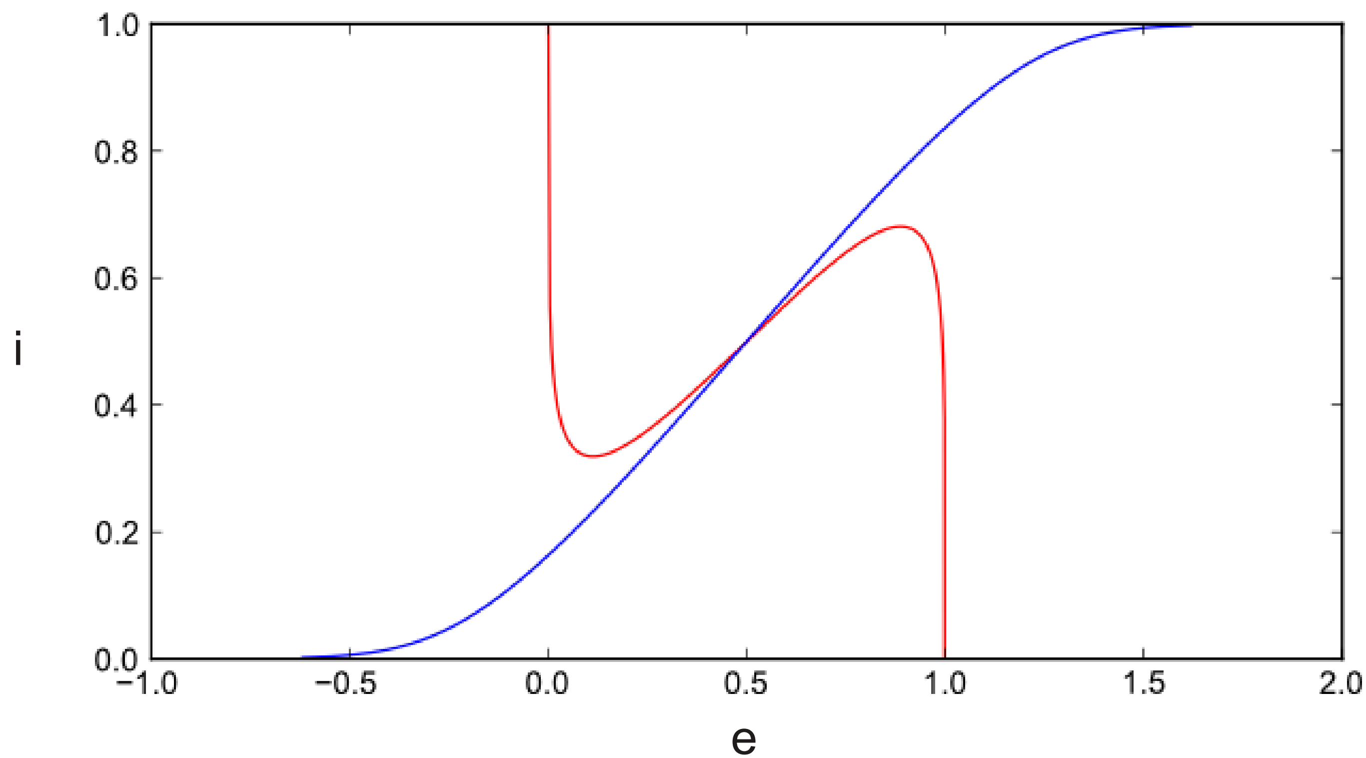
$$\frac{de}{dt}=\frac{1}{\tau_e}\left(-e+S\left(w_{ee}e+w_{ie}i+A_e\right)\right)$$

$$\frac{di}{dt}=\frac{1}{\tau_i}\left(-i+S\left(w_{ei}e+w_{ii}i+A_i\right)\right)$$

With perfect E/I balance, it's unstable!

w_{ee} = 10, w_{ei} = 10 w_{ie}=-10 w_{ii} =-10

A_e = 0 A_i=0



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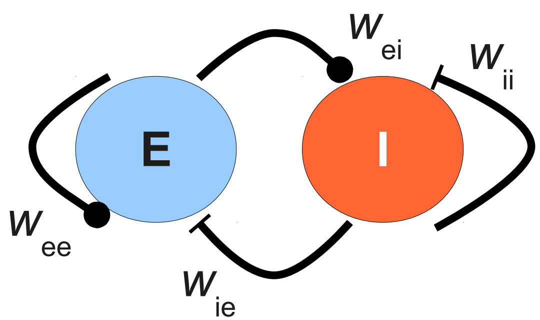

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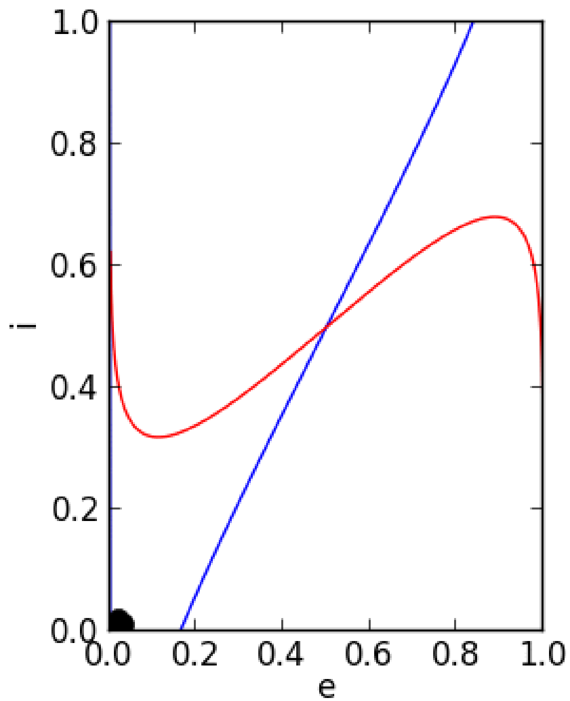

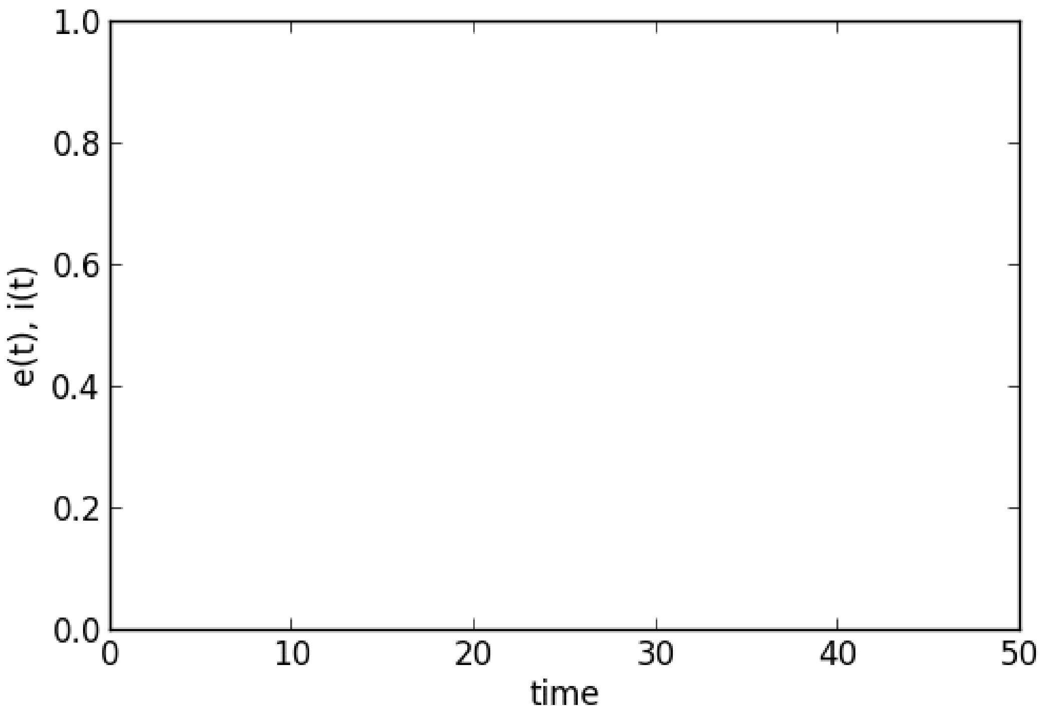

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